A Note on the Averaging Analysis for One-Dimensional Pulsating Detonations

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Abstract

It has been well established, from both experiments and numerical simulations, that gaseous detonation waves are inherently unstable [1]. To elucidate various dynamics of the detonation wave, its unstable structure needs to be fully described. Experimentally, it is challenging to quantitatively measure the detonation structure or even to unambiguously determine the wide spectrum of unstable detonation cell sizes. For practical purposes, there is a need to define at least a length scale to characterize the overall thickness of the unstable cellular front [2, 3]. Despite the fact that gaseous detonations are highly unstable and transient, it is hoped that one can at least define a steady, one-dimensional, average "effective thickness" of the detonation in its direction of propagation. This effective thickness is generally referred to as the "hydrodynamic thickness" of the detonation. Once such a length scale is obtained, it can be used to develop empirical correlations or models for other detonation dynamic parameters (i.e., initiation energy, critical tube diameter, propagation limit, etc.).

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The definition of the hydrodynamic thickness is not unique. Experimentally, attempts have been made to measure the location of the sonic plane from the detachment of the bow shock on a blunt body placed in the path of the detonation [4] and hence to define the hydrodynamic thickness. However, it is difficult to distinguish the detachment distance for the case of unstable cellular detonations. Numerically, recent attempts have been made to simulate a two-dimensional cellular detonation and perform averaging of the flow quantities in order to determine the mean flow field or an equivalent one-dimensional ZND wave [2, 3]. In this manner, the hydrodynamic thickness

is determined from the mean profile at the plane when all fluctuations are equilibrated. Such an averaging approach is found useful and has been used subsequently in many recent numerical studies [5, 6]. Despite further work performed, there remain some issues such as the sampling size invoked and the validity of the different averaging methods.

The unsteady, nonlinear dynamics of the propagation of a one-dimensional pulsating detonation wave are modelled by solutions of the one-dimensional inviscid compressible, time-dependent reactive Euler equations with a single-step irreversible Arrhenius kinetic rate law. These governing equations are solved numerically using a second-order centered scheme, namely the SLIC (Slope Limiter Centered) scheme; its detailed description can be found in [7]. The computational setup follows previous studies by fixing the dimensionless parameters with the values $Q = 50$ and $\gamma = 1.2$ while varying the activation energy Ea as a control parameter [8, 9]. For all the simulations, a numerical grid resolution of 100 points per half reaction zone length L1/2 of the steady ZND detonation is used to ensure the detailed features of the pulsating front are properly resolved. The computations are initialized by the steady solution of the ZND detonation.

In order to obtain the mean structure of the one-dimensional pulsating detonation wave, the computational results are analyzed via a density-weighted (Favre), spatio-temporal averaging method. The averaging analysis was introduced to the field of detonation by Lee & Radulescu [2] and Radulescu et al. [3], and recently employed by Sow et al. [5] and Mi et al. [6]. Two techniques of averaging were considered: (1) using a coordinate moving at the average propagation velocity of the detonation wave and (2) an instantaneous, shock-fixed frame analysis. In a reference frame moving at the average velocity Vavg, the spatial coordinate and particle velocity are transformed as $x\square = x - Vavgt$ and $u\square = u - Vavg$, respectively; in a reference frame moving at the instantaneous propagation velocity Vins, □ and □ are transformed as…